

1. Solve inequality

$$2018 - 1008(2018 - 1008(2018 - \dots - 1008(2018 - 1008x) \dots)) > x$$

If the number of right side brackets is 2018.

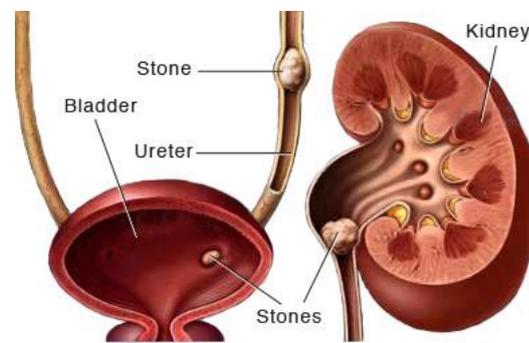
(7 points)

2. Let given polynomial  $P(x)$  with integer coefficients, whereas equation  $|P(x)|=1$  has not less than three different integer roots. Prove that  $P(x)$  has no integer roots. (9 points)

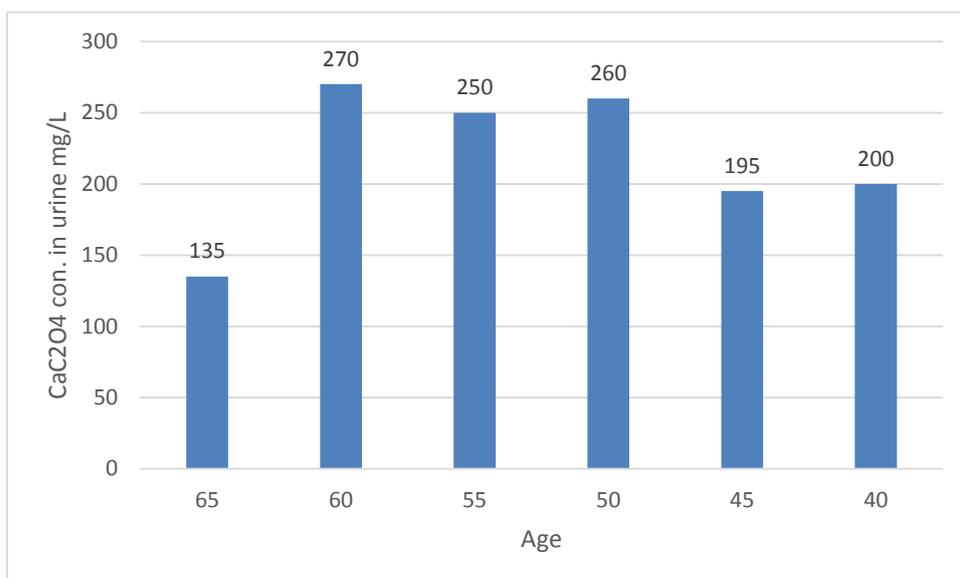
3. Prove that of all the quadrangles described in the outside aroundof the circle, the one which has the smallest perimeter is a square. (11 points)

4. From the point  $O$ , chosen inside of the equilateral triangle  $ABC$  have been drawn perpendiculars  $OM, ON, OP$  to the sides  $BC, CA, AB$  respectively. Prove that the sum of the lengths of the segments  $AP, BM, CN$  does not depend on the position of the point  $O$ . (13 points)

5. Nephrolithiasis is a disease of hard deposition of stones made of minerals and salts that form inside human kidneys. Stones form in the urinary tract when certain chemicals become too concentrated in urine. By far the most common kidney stones are those formed from calcium and oxalate.



(a) The tolerated concentration of calcium oxalate in urine is approximately 0.002 M. At the following graph, calcium oxalate concentration in the patient's urine as a function of patient age groups is demonstrated. At what age/s calcium oxalate concentration is higher that the tolerated concentration?  $MM(\text{CaC}_2\text{O}_4)=128.10 \text{ g/mol}$  (5 points)



(b) Why are patients with  $\text{CaC}_2\text{O}_4$  kidney stones often advised to drink large amounts of water? (2 points)

(c) The calcium and magnesium in a urine sample are precipitated as oxalates. A mixed precipitate of  $\text{CaC}_2\text{O}_4$  and  $\text{MgC}_2\text{O}_4$  resulted and is analyzed by a thermogravimetric procedure. The precipitate mixture is heated to form  $\text{CaCO}_3$  and  $\text{MgO}$ . This second mixture weighed 0.072 g. After ignition to form  $\text{CaO}$  and  $\text{MgO}$ , the resulting solid weighed 0.054 g. What is the mass of  $\text{CaC}_2\text{O}_4$  in the original sample?  $MM(\text{MgC}_2\text{O}_4) = 112.32 \text{ g/mol}$ ,  $MM(\text{CaCO}_3) = 100.09 \text{ g/mol}$ ,  $MM(\text{MgO}) = 40.3 \text{ g/mol}$ ,  $MM(\text{CaO}) = 56.08 \text{ g/mol}$ , (8 points)

6. The Fischer-Tropsch (FT) process is utilized to get sulfur and aromatic free hydrocarbons from syngas. The following reaction is an example to simple FT process:



1.0 mol CO and 3.0 mol  $\text{H}_2$  are poured in 10 dm<sup>3</sup> sealed container at 1400 K. The equilibrium constant ( $K_c$ ) for the reaction is 3.92. Find the equilibrium composition in mol/L of the reaction mixture. (9 points)

7. Four phosphorus atoms are joined in phosphorus molecule ( $\text{P}_4$ ). Supplier commonly provides it in chalk-like cylindrical form. Cylindrical white phosphorus is 5 cm long and 2.1 cm in diameter. Density of the white phosphorus is 1.8 g/cm<sup>3</sup>. Average atomic mass of phosphorus: 31 g/mol

(a) Determine the number of moles of phosphorus molecule present (4 points).

(b) Draw Lewis structure for the phosphorus molecule ( $\text{P}_4$ ) (2 points).

$$V_{\text{cylinder}} = \pi r^2 h$$

8. Electrical water heater has two spirals. Spirals could input to the circuit individually also together parallel and consistently. When switch on both spirals water heated  $t_1 = 6 \text{ min}$  or in other case,  $t_2 = 25 \text{ min}$  dependently on how spirals input to the circuit. Find the times when the spirals included to the electrical circuit individually. Also determine correspondence  $t_1$  and  $t_2$  to parallel and consistently includings of spirals. In all cases, amount of the water in the system and initial temperature is the same. Also lose of the heating is not taken into account. (6 points)

9. A object with 5 kg mass is on the inclined plane. The minimal force for moving up is 15 N and the minimal force for moving down is 10 N on the plane. Find the friction coefficient of the object on the surface of the inclined plane. (4 points)

10. The mechanism, fixed on the middle of the vertical wall of the pool. The pool is full of sea water. The mechanism throws out the objects made of the materials with the density  $\rho_1 = 2400 \text{ kg/m}^3$ ,  $\rho_2 = 1200 \text{ kg/m}^3$  and  $\rho_3 = 800 \text{ kg/m}^3$  with the speed of  $v$  and with the angle  $\alpha = 45^\circ$  up to the horizon. The value of the speed such as, the objects fly from the water and continue moving in air. Resistivity of the water and air are not taken into account. The density of the water is  $\rho_{\text{water}} = 1200 \text{ kg/m}^3$ .



Considering the fact that beginning of the coordinate is located on the throwing point of the objects, define the parameters which characterises the motion and the trajectory of the objects (the coordinates of the points where the object overlooks of the surface of the water and return back inside of the water; velocities and the angles of the vectors of speeds relatively to the horizon in these points; the coordinates of the top points of the trajectory and the maximal deepness where the object can reach after the re-immersion to water) the deepness of the pool is  $h$ , the throw velocity of the object is  $v$ , the acceleration of the free fall is  $g$ , angle  $\alpha$  (directly use  $\alpha = 45^\circ$ ) and corresponding density. Draw the shematical graph of the trajectory of all three the objects. (20 points)